Simple quantum mechanics explains GSI Darmstadt oscillations Even with undetected neutrino; Momentum conservation requires Same interference producing oscillations in initial and final states

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Abstract

GSI experiment on K-capture decay of radioactive ion investigates neutrino masses and mixing without detecting neutrino. States of neutrinos emitted in beta decay include coherent linear combinations of states with different masses, different momenta and same energy. Weak decay described by Fermi Golden Rule conserves momentum but not unperturbed energy. Continuous monitoring collapses wave function and broadens decay width. Initial state before transition also contains coherent linear combination of states with same momentum difference, well defined relative magnitude and phase but broadened energies. One-particle state with a definite momentum difference also has an easily calculated energy difference. Short time between last monitoring and decay allows broadened initial states with different unperturbed energies to decay to final states with single energy. In time interval between creation of ion and decay a linear combination of two states with different unperturbed energies oscillates in time. Measuring oscillation period gives value for difference between squared neutrino masses of two neutrino mass eigenstates. Value obtained from crude approximation with no free parameters for this 'two-slit' experiment in momentum space differs by less than 10% from result observed by KAMLAND. Observing only ion disappearance without detecting neutrino avoids signal suppression by low neutrino absorption cross section.

I. INTRODUCTION

A. The wave function of an unobserved neutrino

A recent experiment [1] describes an oscillation observed in the decay of a radioactive ion before and during the emission of an unobserved neutrino. This phenomenon offers a new and very interesting method for determining neutrino masses and mixing angles [2–4].

To understand oscillations in the time of decay of the initial state, we use energy-momentum conservation to determine the properties of the initial state. Even though the neutrino itself is unobserved the fact that we know it can oscillate tells us that the final state contains a coherent mixture of neutrino mass eigenstates with the same energy and different momenta that is emitted in electron capture decays and called an electron neutrino. Since momentum is conserved in the weak transition that creates the neutrino, the initial ion state must also contain a coherent mixture of two states with the same momentum difference. This property of the initial state is completely independent of whether the neutrino is detected. One-particle ion states with different momenta have different unperturbed energies. But the initial state is repeatedly monitored showing that it has not yet decayed. The time interval between the last evidence for the ion initial state and its decay time is so short that states with different unperturbed energies can decay into the same final state with a single energy. The relative phase between two states with different unperturbed energies changes with time and produces the observed oscillations.

We now calculate this energy difference and the period of oscillations.

B. Momentum conservation determines path from oscillating ν to mother ion

The weak decay is a transition between the initial "mother" ion wave packet to a final state containing a recoil "daughter" ion with a definite energy and momentum and a neutrino wave packet which contains states with different neutrino mass, different momenta and the same energy. This decay is described by first order time-dependent perturbation theory [5].

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The transition from an initial state denoted by $|i(t)\rangle$ to a given final state denoted by $|f\rangle$ is given by Fermi's Golden Rule. The transition probability per unit time at time t is

$$W(t) = \frac{2\pi}{\hbar} |\langle f| T |i(t)\rangle|^2 \rho(E_f) \tag{1.1}$$

where $\langle f| T |i(t)\rangle$ denotes transition matrix element determined in this case by weak interaction theory. This treatment shows that momentum is conserved. The result that energy conservation is violated at short times is confirmed experimentally by the broadening of decay widths at short time. This broadening is important for the understanding of the oscillations.

Consider a component of the initial mother wave packet which has a momentum $\vec{P}$ and energy E_i. The final state has a recoil ion with momentum denoted by $\vec{P}_R$ and energy E_R and a neutrino with mass m, energy E_ν and momentum $\vec{p}_\nu$. We assume conservation of momentum, but that energy is not conserved because of the short time between the last monitoring and observation of the final state. The energy of the final state is denoted by $E_f \neq E_i$. The conservation laws then require

$$E_R = E_f - E_\nu; \quad \vec{P}_R = \vec{P} - \vec{p}_\nu; \quad (E_f - E_\nu)^2 - (\vec{P} - \vec{p}_\nu)^2 = M_R^2 \tag{1.2}$$

$$E_f^2 + E_\nu^2 - \vec{P}^2 - \vec{p}_\nu^2 = E_f^2 - E_i^2 + M^2 + m^2 = M_R^2 + 2E_f E_\nu - 2\vec{P} \cdot \vec{p}_\nu \tag{1.3}$$

$$\Delta(m^2) \approx (E_i + E_f)(\delta E_i - \delta E_f) + 2E_f \delta E_\nu - 2P\delta p_\nu \approx E_f \delta E_i + (E_f - E_i)\delta E_f - 2P(\delta P) \tag{1.4}$$

$$\frac{\Delta(m^2)}{2P\delta P} = \frac{\Delta(m^2)}{2E_i \delta E_i} \approx \frac{E_f}{2E_i} + \frac{(E_f - E_i)\delta E_f}{2E_i \delta E_i} - 1 \approx \frac{E_f - E_i}{2E_i} \cdot \left[1 + \frac{\delta E_f}{\delta E_i}\right] - \frac{1}{2} \approx -\frac{1}{2} \tag{1.5}$$

Where we have noted that the small violation of energy conservation $(E_f - E_i) \ll E_i$

C. The period of oscillation

The phase difference at a time t between states produced by the neutrino mass difference on the motion of the initial ion in the laboratory frame is

$$\delta\phi \approx -\delta E_i \cdot t = \frac{\Delta(m^2)}{E_i} = \frac{\Delta(m^2)}{\gamma M} \tag{1.6}$$

where γ denotes the Lorentz factor E/M. The period of oscillation δt is obtained by setting $\delta\phi \approx -2\pi$,

$$\delta t \approx \frac{2\pi E_i}{\Delta(m^2)} = \frac{2\pi\gamma M}{\Delta(m^2)} \tag{1.7}$$

The previously obtained [2] theoretical value for $\Delta(m^2)$, denoted by $\Delta(m^2)_{Kienle}$, differs from ours (1.7) denoted by $\Delta(m^2)_{HJL}$

$$\Delta(m^2)_{Kienle} = \frac{4\pi\gamma M}{\delta t} \approx 2.75\Delta(m^2)_{exp}; \quad \Delta(m^2)_{HJL} \approx \frac{\Delta(m^2)_{Kienle}}{2} \approx 1.37\Delta(m^2)_{exp} \tag{1.8}$$

where the value of $\Delta(m^2)_{exp}$ is the value obtained from neutrino oscillation experiments [2]

That our theoretical value for $\Delta(m^2)$ obtained with minimum assumptions and no free parameters is so close to the experimental value obtained from completely different experiments suggests that better values obtained from better calculations can be very useful in determining the masses and mixing angles for neutrinos.

II. DETAILS OF THE K-CAPTURE EXPERIMENT

A radioactive nucleus in an ion decays by capturing an electron from the K-shell or other atomic shell and emits a monoenergetic neutrino. The emitted electron-neutrino ν_e is a linear combination of several neutrino mass eigenstates. If the initial state has a definite momentum and energy and if energy and momentum are conserved, the energy and momentum of the neutrino are determined and therefore its mass. This would then be a missing mass" experiment in which the mass of the neutrino is determined without the observation of the neutrino. Interference between amplitudes from different neutrino mass states cannot be observed in such a missing-mass experiment. The experimental observation that interference actually occurs shows that this cannot be a missing mass experiment. Energy is not conserved because of the short time between the last time when the ion was observed to have not yet decayed and the decay time.

It may seem rather peculiar that neutrino oscillations can be observed in the state of a radioactive ion before its decay into an unobserved neutrino. One wonders about causality and how the initial ion can know how it will decay. But much discussion and thought revealed that the essential quantum mechanics is a "two-slit" or "which-path" experiment [6] in momentum-energy space. Causality is preserved because no information about the final state is available to the initial ion.

A. Actual measurement in observation of decay is not generally understood

- The ion is monitored at regular intervals during passage around the storage ring.

- Each monitoring collapses the wave function (or destroys entanglement phase).

- Time in the laboratory frame is measured at each wave function collapse.

- The the decay of the initial state is observed by the disappearance of the ion between successive monitorings.

Repeated monitoring by interactions with laboratory environment at regular time intervals and same space point in laboratory collapses wave function and destroys entanglement [7] First-order time dependent perturbation theory gives probability for initial state decay during small interval between two monitoring events. Final amplitudes completely separated at long times have broadened energy spectra overlapping at short times. Their interference produces oscillations between Dicke superradiant [8] and subradiant states having different transition probabilities.

Experiment measures momentum difference between two contributing coherent initial states and obtains information about ν masses without detecting ν. Simple model relates observed oscillation to squared ν mass difference and gives value differing by less than 50% from values calculated from KAMLAND experiment. Monitoring simply expressed in laboratory frame not easily transformed to other frames and missed in Lorentz-covariant descriptions based on relativistic quantum field theory.

The initial ion wave function is a wave packet containing a combination of energies and momenta. The weak decay transition then produces a recoiling ion and a final neutrino in a coherent mixture of its mass eigenstates. It is not a missing mass experiment because the energies of the components of the initial state wave function were not measured and nothing about the final neutrino state was measured.

B. Oscillations are produced on an initial state even without neutrino detection

1. Oscillations in time can occur only if there is interference between two components within the initial state wave function with different energies.

   - The initial state has a wave function with a definite mass

   - Interferent between components of the initial wave function with different energies must have different momenta.

   - If energy and momentum are conserved in the transition the final state must also have components with both different energies and different momenta.

2. Which components of the unmeasured final state are coherent?

   - In ordinary neutrino oscillations the detector chooses coherent components with the same energy and different momenta

   - Here there is no detector. Any coherent final state must be mixturew of both energy and momentum

3. Coherence occurs between components of the final ν_e state with different masses, momenta and energies but the same velocity.

   - Components of a ν_e state with different masses, momenta and energies but the same velocity remain a ν_e state forever

   - Coherence arises when components of an initial state have the same momentum and energy differences as the components a ν_e state having the same velocity

Since the same final ν_e state can be produced by any of the momentum components in the initial wave function, the path in energy-momentum space between the initial and final states is not known and the corresponding amplitudes can be coherent and interfere.

The relative phases in the initial wave function are determined by its localization in space at the point of entry into the apparatus. These relative phases change with time in accordance with the relative energy differences in the packet. They are independent of the final state, which is created only at the decay point. Thus there is no violation of causality. ν_e from several mass eigenstates depends upon the relative phases of the contributions from components in the inital wave function having different energies and momenta. These relative phases increase linearly with time and produce oscillations.

Observing the period of these oscillations gives information about the neutrino mass differences and the mixing angles of the neutrino mass matrix. Reliable detailed values for the relation between the observed oscillation period and neutrino mass differences are not obtained in the crude models so far considered. At this point the fact that the value obtained (1.7) is so close to values obtained from neutrino oscillation experiments is encouraging.

C. Dicke superradiance and subradiance in the experiment

The initial radioactive "Mother" ion is in a one-particle state with a definite mass moving in a storage ring. There is no entanglement [7] since no other particles are present. The final state denoted by $|f(E_\nu)\rangle$ has a "daughter" ion and an electron neutrino ν_e which is a linear combination of two neutrino mass eigenstates denoted by ν_1 and ν_2 with masses m_1 and m_2. To be coherent and produce oscillations the two components of the final wave function must have the same energy E_ν for the neutrino and the same momentum $\vec{P}_R$ and energy E_R for the "daughter" ion.

$$|f(E_\nu)\rangle \equiv \left|\vec{P}_R; \nu_e(E_\nu)\right\rangle = \left|\vec{P}_R; \nu_1(E_\nu)\right\rangle \langle \nu_1 | \nu_e \rangle + \left|\vec{P}_R; \nu_2(E_\nu)\right\rangle \langle \nu_2 | \nu_e \rangle \qquad (2.1)$$

where $\langle \nu_1 | \nu_e \rangle$ and $\langle \nu_2 | \nu_e \rangle$ are elements of the neutrino mass mixing matrix, commonly expressed in terms of a mixing angle denoted by θ.

$$\cos\theta \equiv \langle\nu_1|\nu_e\rangle; \quad \sin\theta \equiv \langle\nu_2|\nu_e\rangle; \quad |f(E_\nu)\rangle = \cos\theta\left|\vec{P}_R;\nu_1(E_\nu)\right\rangle + \sin\theta\left|\vec{P}_R;\nu_2(E_\nu)\right\rangle \quad (2.2)$$

We use a simplified two-component initial state for the "mother" ion having two components $\left|\vec{P},E\right\rangle$ and $\left|(\vec{P}+\delta\vec{P}),(E+\delta E)\right\rangle$. Since the states $\nu_1(E_\nu)$ and $\nu_2(E_\nu)$ have the same energies and different masses, they have different momenta. After a very short time two components with different initial state energies can decay into a final state which has two components with the same energy and a neutrino state having two components with the same momentum difference $\delta\vec{P}$ present in the initial state.

The momentum conserving transition matrix elements between the two initial momentum components to final states with the same energy and momentum difference $\delta\vec{P}$ are

$$\langle f(E_\nu)|\,T\,\big|\vec{P}\big\rangle = \cos\theta\left\langle\vec{P}_R;\nu_1(E_\nu)\big|\,T\,\big|\vec{P}\right\rangle; \quad \langle f(E_\nu)|\,T\,\big|\vec{P}+\delta\vec{P}\big\rangle = \sin\theta\left\langle\vec{P}_R;\nu_2(E_\nu)\big|\,T\,\big|\vec{P}+\delta\vec{P}\right\rangle$$

$$(2.3)$$

The Dicke superradiance [8] analog here is seen by defining superradiant and subradiant linear combinations of these states

$$|Sup(E_\nu)\rangle \equiv \cos\theta\,|P)\rangle + \sin\theta\,|P+\delta P)\rangle; \quad |Sub(E_\nu)\rangle \equiv \cos\theta\,|P+\delta P)\rangle - \sin\theta\,|P)\rangle \quad (2.4)$$

The transition matrix elements for these two states are then

$$\frac{\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle}{\langle f|\,T\,|P\rangle} = [\cos\theta + \sin\theta]; \quad \frac{\langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle}{\langle f|\,T\,|P\rangle} = [\cos\theta - \sin\theta] \quad (2.5)$$

where we have neglected the dependence of the transition operator T on the small change in the momentum P. The squares of the transition matrix elements are

$$\frac{|\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle|^2}{|\langle f|\,T\,|P\rangle|^2} = [1 + \sin 2\theta]; \quad \frac{|\langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle|^2}{|\langle f|\,T\,|P\rangle|^2} = [1 - \sin 2\theta] \quad (2.6)$$

For maximum neutrino mass mixing, $\sin 2\theta = 1$ and

$$|\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle|^2 = 2|\langle f|\,T\,|P\rangle|^2; \quad |\langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle|^2 = 0 \quad (2.7)$$

This is the standard Dicke superradiance in which all the transition strength goes into the superradiant state and there is no transition from the subradiant state.

Thus from eq. (2.4) the initial state at time t varies periodically between the superradiant and subradiant states.

D. How a "watched pot experiment" can give a nonexponential decay

The experiment observes a time-dependence in the decay probability which is not exponential. Although this appears at first to be counterintuitive, it follows naturally from a crucial feature of being a "watched pot" experiment. The initial state of the ion is monitored during its passage around a storage ring, thereby affirming that the ion has not yet decayed. It is like the "Schroedinger cat" experiment in which the door is always open so that there is a continuous measurement of whether the cat is still alive.

The wave function describes the motion of the initial state as a free ion moving in the fields of the apparatus for a time t defined as the time interval between its entry into the apparatus and the last time before the decay in which it was affirmed not to have decayed. The time t' between the last monitoring and the time of decay is negligible for the purpose of the analysis of the experiment.

$$t' \ll t \tag{2.8}$$

The initial state denoted by $|i\rangle$ is a wave packet containing components with different energies. The relative phases of these components in the initial wave function are determined by its localization in space at the point of entry into the apparatus. The changes with time of these relative phases are described by a Hamiltonian denoted by H_o which describes the motion of a free initial ion moving in the electromagnetic fields constraining its motion in a storage ring. The wave function describing the evolution of the initial state in time is thus

$$|i(t)\rangle = e^{iH_o t} |i\rangle \tag{2.9}$$

The decay transition to a given final state denoted by $|f\rangle$ is described by a transition matrix element

$$\langle f| \, T \, |i(t)\rangle = \langle f| \, T e^{iH_o t} \, |i\rangle \tag{2.10}$$

The transition probability per unit time at time t is given by Fermi's Golden Rule,

$$W(t) = \frac{2\pi}{\hbar} |\langle f | T | i(t) \rangle|^2 \rho(E_f) = \frac{2\pi}{\hbar} |\langle f | T e^{iH_o t} | i \rangle|^2 \rho(E_f) \qquad (2.11)$$

We can now see why the time dependence of the decay is not exponential. The probability P_i that the ion is still in its initial state and has not yet decayed satisfies the differential equation

$$\frac{d}{dt} P_i = -W(t) P_i; \qquad \frac{d}{dt} log(P_i) = -W(t). \qquad (2.12)$$

Solving this equation gives

$$P_i = e^{-\int W(t) dt} \qquad (2.13)$$

We see immediately why decays are generally exponential and this one need not be. Usually the transition matrix element (2.10) and the transition probability (2.11) are independent of time and eq. (2.13) gives an exponential decay. Here the transition probabilty depends upon the propagation of the initial state during the time t between the entry of the ion into the apparatus and the time of the decay.

For a simple gedanken example consider the decay of a spin-1/2 radioactive particle with a spin-dependent interaction which allows it to decay with a lifetime of five days but only when the spin is polarized in the $+x$ direction. The decay is forbidden from the $-x$ state. Consider a beam of such particles moving in the z-direction, polarized at time $t = 0$ in the $+x$ direction with a weak magnetic field in the z-direction causing the spin to precess with a period of seven seconds around the z-axis. The decay rate will not be exponential but will be modulated by a periodic function with a period of seven seconds.

Since the time dependence depends only on the propagation of the initial state, it is independent of the final state, which is created only at the decay point. Thus there is no violation of causality. No information about the final state exists before the decay. Although time-dependent perturbation theory might suggest that a decay amplitude can be present before the decay, the continued observation of the initial ion before the decay rules out any influence of any final state amplitude on the decay process.

E. A tiny energy scale

The experimental result, if correct, sets a scale in time of seven seconds, which means a tiny energy scale for the difference between two waves which beat with a period of seven seconds.

$$\Delta E \approx 2\pi \cdot \frac{\hbar}{7} = 2\pi \cdot \frac{6.6 \cdot 10^{-16}}{7} \approx 0.6 \cdot 10^{-15} \text{eV} \tag{2.14}$$

This tiny energy scale cannot come out of thin air. It must be predictable from standard quantum mechanics using a scale from another input. The only other input available according to eq. (2.13) is in the propagation of the initial state through the storage ring during the time interval between the entry into the apparatus and the decay. One tiny scale available in the parameters that describe this experiment is the mass-squared diffgference between two neutrino mass eigenstates. This gives a tiny mass scale when this mass-squared is divided by the energy of the ion.

$$\frac{\Delta(m^2)}{E} \approx \frac{0.8 \cdot 10{-4}}{3 \cdot 10^{11}} \approx 2.7 \cdot 10^{-15} \text{eV} \tag{2.15}$$

where the value of $\Delta(m^2)$ is obtained from neutrino oscillation experiments [2].

That these two tiny energy scales obtained from completely different inputs are within an order of magnitude of one another suggests that they must be related by a serious quantum-mechanical calculation. The simplest model relating these two tiny mass scales gives a result that differs only by 10%. The fact that the observed seven second period creates a tiny mass scale and that no other energy in this experiment comes even orders of magnitude close to this scale suggests that this is not an accident. These two scales appear in the analysis of the same experiment where there must be a theoretical prediction for the seven second scale if we believe quantum mechanics.

There are many other possible mechanisms for producing oscillations. The experimenters [1] claim that they have investigated all of them. We also note that all these other mechanisms involve energy scales very different from the scale producing a seven second period.

III. THE TWO PRINCIPAL DIFFICULTIES OF NEUTRINO EXPERIMENTS

1. Ordinary neutrino oscillation experiments are difficult because

   - The neutrino absorption cross section is tiny. The number of neutrino events actually used in ordinary experiments is many orders of magnitude smaller than the number events creating the neutrinos.

   - The oscillation wave lengths are so large that it is difficult to actually follow even one oscillation period in any experiment.

2. This experiment opens up a new line for dealing with these difficulties

   - The oscillation is measured without detecting the neutrino. Detection of every neutrino creation event avoids the losses from the low neutrino absorption cross section.

   - The long wave length problem is solved by having the radioactive source move a long distance circulating around in a storage ring. The data if correct show many oscillations in the same experiment.

This paper considers the basic quantum mechanics of the first difficulty and shows in a crude approximation that it is possible in principle to observe and measure neutrino oscillations by looking only at the radioactive source.

The theoretical analysis in this paper was motivated by discussions with Paul Kienle at a very early stage of the experiment in trying to understand whether the effect was real or just an experimental error.

IV. CONCLUSIONS

A new oscillation phenomenon providing information about neutrino mixing is obtained by following the initial radioactive ion before and during the decay. The difficulties introduced in conventional neutrino experiments by the tiny neutrino absorption cross sections

and the very long oscillaton wave lengths are avoided here. Measuring the decay time enables every neutrino event to be observed and counted without the necessity of observing the neutrino via the tiny absorption cross section. The confinement of the initial ion in a storage ring enables long wave lengths to be measured within the laboratory.

Coherence between amplitudes produced by the weak decay of a radioactive ion by the emission of neutrinos with different masses has been shown to follow from the localization of the initial radioactive ion within a space interval much smaller than the oscillation wave length. This coherence is observable in following the motion of the initial radioactive ion from its entry into the apparatus to its decay.

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